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A FAST "STRETCHED SPACE" METHOD FOR GENERATING SYNTHETIC VERTICAL SHEETS OF NON-STATIONARY STOCHASTIC ATMOSPHERIC STRUCTURE

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17 March 1995

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Geophysical phenomena are often characterized by known or assumed power spectral density functions that have an asymptotic negative power law slope dependency. Various filter synthesis methods are rountinely employed to generate synthetic scenes from pseudorandom arrays by passing stochastic data through filters having a desired correlation structure and power spectral dependency.

This report examines the possibility of producing a two-dimensional vertical sheet of non-stationary synthetic structure by means of a "stretched space" transformation. Since computations that apply multidimensional transforms to large data arrays consume enormous computer resources and run times, the goal of this study is to find an alternate method to reduce the heavy computational burden. Future editions of the Phillips Laboratory Strategic High Altitude Atmospheric Radiance Code (SHARC) will feature an ability to calculate structured radiance. The method explored herein provides a process for rapidly generating large 2-D arrays of non-stationary structure.

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Contents

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A Fast "Stretched Space" Method for Generating Synthetic Vertical Sheets of Non-Stationary Stochastic Atmospheric Structure

1. INTRODUCTION

Atmospheric fluctuations in wind speed, temperature, and density can be characterized by continuous power spectral density functions. Such spectra, parameterized by spectral slope, variance, and correlation lengths are often used in simulating an environment or predicting atmospheric structure. Multidimensional fast Fourier transform synthesis provides a means for filtering white noise with spatial filters to simulate a stationary time or spatial data set. Autoregression synthesis provides a fast means for simulating a one-dimensional *non-stationary* spatial structure sequence.

The Phillips Laboratory Strategic High Altitude Atmospheric Radiance Code (SHARC)[1] uses first principles to calculate point-to-space and limb-viewing atmospheric background infrared (IR) radiance and transmittance under both local-thermal-equilibrium (LTE) and non-local-thermal-equilibrium (NLTE) conditions above 50 km. Release 3 of the SHARC code[2] predicts IR radiation and transmittance in the 1-40 μm spectral region and includes important bands from the major isotopes of NO, CO, H_2O, O_3, OH, CO_2, CH_4, and NO^+. Specific local atmospheric environments can be specified through region definitions, and diurnal characteristics can be specified through user or program generated multiple vertical concentration profiles. A subroutine module breaks a given line-of-sight (LOS) specification into small segments and determines the composition and properties of each segment. Each segment is determined by the intersection of the LOS with an altitude layer boundary, defined by input atmospheric profiles. Appropriate profiles of temperature, pressure, and molecular state densities are determined for each segment.

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[1]Sharma, R.D., Gruninger, J.H., Sundberg, R.L., Duff, J.W., Bernstein, L.S., Robertson, D.C., and Healey, R.J., (1991) *Description of SHARC-2, The Strategic High Altitude Atmospheric Radiance Code*, Phillips Laboratory technical report, PL-TR-91-2071, ADA 239 008

[2]Gruninger, J., Sundberg, R.L., Duff, J.W., Bernstein, L.S., Matthew, M.W., Adler-Golden, S., Robertson, D., Sharma, R., Brown, J.H., Healey, R., and Vail J., (1994) SHARC - 3, A Model for Infrared Radiance at High Altitudes, *Proceedings SPIE - The International Society of Optical Engineering*, V2223, April 1994, Orlando, Florida.

A future release of SHARC[3] will have the ability to provide realizations of atmospheric infrared volume-emission perturbations that occur from fluctuations in temperature and density of the contributing molecular species. Version 4 of the SHARC code envisions a capability to evaluate radiance structure from estimated variances in the standard temperature and density profiles. The algorithms will simulate IR fluctuations that must depend on relatively small fluctuations in atmospheric species number densities, vibrational state populations, and the kinetic temperatures along a given line-of-sight[4]. Where NLTE effects dominate, (generally above 50 km) a small fluctuation in kinetic temperature can produce correlated, anti-correlated, or no change in the vibrational state temperature. Such changes ultimately depend on the relative contributions from total number density, temperature-dependent kinetic rates, and radiative relaxation. A proper description of the temperature/density/radiance field as viewed from a point sensor is thus needed to enable SHARC to correctly compute the radiance structure field.

To provide a realistic but practical two-dimensional structure scene capability requires a creative and efficient algorithm. This report presents a method for producing a two-dimensional vertical sheet of atmospheric non-stationary synthetic stochastic structure by means of a "stretched space" mapping transformation. The benefit of the transform is that it requires just a single filter pass in stationary space to generate the entire structure and so reduces the computation time by the order of the array. This report expands upon the results expressed in three previous reports that dealt with one-dimensional autoregression[5], two-dimensional autoregression/moving average[6], and three-dimensional hybrid[7] structure simulation.

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[3]Sundberg, R.L., Gruninger, J., De, P., and Brown, J..H. (1994) Infrared Radiance Fluctuations In The Upper Atmosphere, *Proceedings SPIE - The International Society of Optical Engineering*, V2223, April 1994, Orlando, Florida.

[4]Sears, R.D., Strugala, L.A., Newt, J., Robertson, D., Brown, J.H., Sharma, R., (1994) Simulation of the Infrared Structured Earthlimb Background Using the SHARC Radiance Code, 32nd Aerospace Sciences Meeting and Exhibit, Reno, NV, January 1994.

[5]Sundberg, R.L., Gruninger, J., De, P., and Brown, J.H., (1994) Infrared Radiance [5]Brown, J.H., (1993) *Atmospheric Structure Simulation: An Autoregressive Model for Smooth Geophysical Power Spectra with Known Autocorrelation Function*, Phillips Laboratory technical report, PL-TR-2185, ERP#1128, ADA 276691.

[6]Brown, J.H., (1993) *Atmospheric Structure Simulation: An ARMA Model for Smooth Isotropic Two-Dimensional Geophysical Power Spectra*, Phillips Laboratory technical report, PL-TR-93-2224, ERP#1132, ADA 280476.

[7]Brown, J.H., (1994) *Synthetic 3-D Atmospheric Temperature Structure: A Model for Known Geophysical Power Spectra Using a Hybrid Autoregression and Fourier Technique*, Phillips Laboratory technical report PL-TR-94-2150, ERP#1150, ADA 289058.

2. *PSD* MODEL DISCUSSION

Atmospheric power spectral density functions often are modeled by three parameter isotropic one-dimensional double-sided power law functions of the form[8,9]:

$$PSD(k)=\frac{\sigma^2 a^{2\nu}\Gamma(\nu+1/2)}{\sqrt{\pi}\,\Gamma(\nu)\left(a^2+k^2\right)^{\nu+1/2}}, \tag{1}$$

Here k represents spatial frequency, σ^2 is the variance, ν determines the asymptotic power law dependence, and a is a parameter that determines the "low frequency" PSD shape. The relationship between the frequency domain *PSD* and the time or spatial domain autocorrelation function is specified by their Fourier transform pairs. Thus the autocorrelation function, (*ACF*), for the real even *PSD* function is[5] :

$$ACF(s)=\frac{\sigma^2 2^{(1-\nu)}(2\pi\ as)^{\nu}K_\nu(2\pi\ as)}{\Gamma(\nu)}, \tag{2}$$

where K_ν is the Bessel function of the second kind of fractional order. In this form the *PSD* and *ACF* are stationary and independent of altitude The parameter "a" can be expressed in terms of the integral scale, L, of the autocorrelation function such that $a=\dfrac{\Gamma(\nu+1/2)}{2\sqrt{\pi}\,\Gamma(\nu)*L_c}$. For the moment we assume "a" is a constant and "L_c" is a measure of the "correlation length" ‡. The above may be rewritten as:

$$ACF(s)=\frac{\sigma^2 2^{(1-\nu)}}{\Gamma(\nu)}\left(\frac{2\pi s\left[\Gamma\left(\nu+\frac{1}{2}\right)\right]}{2\sqrt{\pi}\,\Gamma(\nu)L_c}\right)^{\nu}K_\nu\left(\frac{2\pi s\left[\Gamma\left(\nu+\frac{1}{2}\right)\right]}{2\sqrt{\pi}\,\Gamma(\nu)L_c}\right) \tag{2}$$

or, $$ACF(s)=B\left(C\frac{s}{L_c}\right)^{\nu}K_\nu\left(C\frac{s}{L_c}\right) \tag{3}$$

where $B=\dfrac{\sigma^2 2^{(1-\nu)}}{\Gamma(\nu)}$ and, $C=2\pi\left(\dfrac{2\sqrt{\pi}\,\Gamma(\nu)}{\Gamma(\nu+1/2)}\right)^{-1}$.

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[8]Tatarski, V.I., (1961) *Wave Propagation in a Turbulent Medium*, Eq. 1.11, McGraw-Hill.

[9]Futterman, W.I., Schweitzer, E.L., Newt, J.E., (1991) Estimation of Scene Correlation Lengths, Characterization, Propagation, and Simulation of Sources and Backgrounds, *Proceedings SPIE - The International Society of Optical Engineering*, V1486, pp127-140, April 1991, Orlando, Florida.

‡ Reference 5 describes the correlation length as the equivalent width of the autocorrelation function, pp. 5-6.

3. THE *"STRETCHED SPACE"* TRANSFORMATION

The general definition of the autocorrelation sequence of the indexed families of "real" random variables, $\{x_n\}$ and $\{x_m\}$, of a probabilistic process is[10]:

$$ACS_{xx}(n,m) = E[x_n x_m] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{x}_n \mathbf{x}_m p_{x_n x_m}(\mathbf{x}_n, n, \mathbf{x}_m, m) d\mathbf{x}_n d\mathbf{x}_m$$

where E is the expected value, $p_{x_n x_m}(\mathbf{x}_n, n, \mathbf{x}_m, m)$ is the joint probability density function, and where the bold letters correspond to the dummy variables of the probability density function. Since the autocovariance function with zero mean is identical to the autocorrelation function, and with index m corresponding to z and index m corresponding to s, the autocovariance function becomes,

$$ACF(z,s) = E[f(z)f(z+s)] \tag{4}$$

where f is a continuous range of random values. Now suppose the *PSD* and *ACF* are stationary and correspond to fluctuations of atmospheric temperature in the direction, z. Under "stationary" conditions, the autocovariance function is not a function of "z" and we can therefore write: $ACF(s) = E[f(z)f(z+s)]$. For the stationary condition, we assume that $f(z)$ is ergodic and we can estimate $ACF(s)$ by:

$ACF(s) = \frac{1}{2p}\int_{-p}^{p}\{f(z)f(z+s)\}dz$, when $f(z)$ is continuous or,

$ACF(s) = \frac{1}{2N+1}\sum_{i=-N}^{N}\{f(z_i)f(z_i+s)\}$, when $f(z)$ is discrete, where $z_i = i * \Delta z$, with Δz the spacing between the z_i levels.

We now wish to examine the case where z corresponds to altitude and $ACF(z,s) = E[f(z)f(z+s)]$ is non-stationary. We propose to transform the lag "s" to the new lag variable "y" by the monotonic function: $y = y(z,s)$ which has an inverse transform $s(z,y) = y^{-1}(z,s)$. We also will transform the altitude "z" to a space "t" by the monotonic function: $t = t(z)$, where y has uniform spacing in t. These transforms provide the stationary autocovariance function $ACF(y)$:

$$ACF(y) = E[f(t)f(t+y)] \tag{5}$$

Now, let us consider the case where the parameter L_c in Eq. (2) is no longer constant but depends on altitude z. If L_c depends on z, then the parameter "a" in Eq. (1) depends upon z, which of course means the *PSD* is non-stationary. Rewriting Eq. (3) for a "correlation length" $L_c(z,s)$ that depends upon the altitude z and lag s, the non-stationary autocovariance function $ACF(z,s)$ becomes:

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[10] Oppenheim, A.V., and Schafer, R.W., (1975), *Digital Signal Processing*, Chapter 8, Prentice-Hall, New Jersey.

$$ACF(z,s) = B\left(C\frac{s}{L_c(z,s)}\right)^{\nu} K_{\nu}\left(C\frac{s}{L_c(z,s)}\right).$$

We assume that the "measured" parameter "a" in the equation for the PSD is an average over altitude, so that $a(z)$ may be defined as an average of the reciprocal of L_c: $a(z) \propto \frac{1}{L_c(z,s)} = \frac{1}{s}\int_z^{z+s} \frac{dx}{L_c(x,0)}$, where $L_c(x,0)$ is a function of altitude.

Now, defining the transformation of s to y by:

$$y = y(z,s) = \overline{L_c}\int_z^{z+s} \frac{dx}{L_c(x,0)} \tag{6}$$

where $\overline{L_c}$ is a constant that will be set for convenience, then, $ACF(y) = B\left(\frac{C}{\overline{L_c}}y\right)^{\nu} K_{\nu}\left(\frac{C}{\overline{L_c}}y\right)$ is stationary and not a function of altitude. In Eq. (6) we constrain L_c to positive values so that $y = y(z,s)$ is monotonic. We also assume that ν is a constant independent of altitude.

Turning our attention to the transformation of z to $t = t(z)$, we set Eq. (4) equal to Eq. (5), or

$$\begin{aligned} ACF(z,s) &= ACF(y) \\ E[f(z)f(z+s)] &= E[f(t)f(t+y)] \end{aligned} \tag{7}$$

Equation (7) is satisfied in general when

$$\begin{aligned} f(z) &= f(t) \\ f(z+s) &= f(t+y) \end{aligned} \tag{8}$$

Equations (8) are true for $t(z) = t$ and $t(z+s) = t + y = t(z) + y$. To construct the transformation, we arbitrarily set $z_0 = t_0 = t_0(z_0) = 50\,\text{km}$. Any other z value is found from $t[z_0 + s(z_0, y)] = t_0 + y$. The transformation $t[z + s(z,y)] = t(z) + y$ may be used for all z values since, we have for $z = z_1$, $t(z_1) = t_0 + y(z_0, z_1 - z_0)$, and, in general,

$t(z) = t_0 + y(z_0, z - z_0) = t_0 + \overline{L_c}\int_{z_0}^{z_0+(z-z_0)} \frac{dx}{L_c(x,0)}$ or,

$$t(z) = t_0 + \overline{L_c}\int_{z_0}^{z} \frac{dx}{L_c(x,0)} \tag{9}$$

We have arbitrarily set $t_0 = z_0 = 50$ km. We would also like t = 300 km when z = 300 km so that the range in z is the same as the range in t. The limits determine $\overline{L_c}$ which is found from

$$\int_{50}^{300} dt = \int_{50}^{300} \frac{\overline{L_c}}{L_c(z,0)} dz \text{ or } \overline{L_c} = \frac{250}{\int_{50}^{300} \frac{1}{L_c(z,0)} dz}.$$

Thus the real space altitude, "z" can be "mapped" into a stretched space, "t(z)" where the auto-covariance function ACF (y) is stationary. Given a positive function, $L_c(z)$ (see for example reference 7), Figure 1 plots "stretched space", t, as a monotonically increasing function of altitude, z. In practice, the increments in t were equally spaced by 0.244 km.

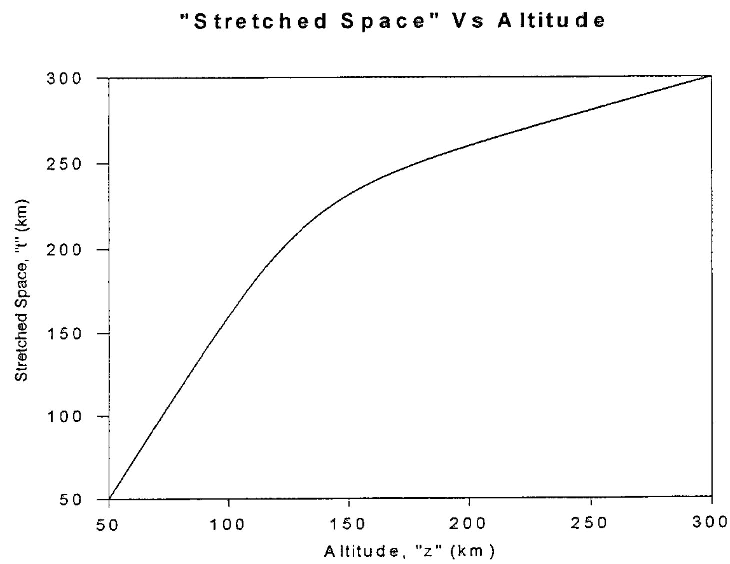

Figure 1. "Stretched-space" parameter, t, plotted as a function of altitude, z.

4. RESULTS

The following discussion is aimed at producing a practical and efficient two-dimensional sheet of non-stationary stochastic atmospheric structure consistent with having pre-assigned vertical and horizontal power spectral densities and autocorrelation functions. In this section, we compute the structure in "stretched" space, map back to "normal" space, and then examine and compare it to the prescribed structure. Correlation lengths and σ^2 variances were taken from the data of Reference 4 and fitted to monotonic "logistic dose response" curves of the form

$y = a + b/\left(1+(x/c)^d\right)$.

A variety of digital filter synthesis methods may be employed to convert a series of Gaussian random numbers to correlated stochastic arrays. Perhaps the most frequently employed technique is the Fourier technique where Gaussian numbers are Fast Fourier Transformed, multiplied by a suitable filter function, and inverse Fast Fourier Transformed back to Cartesian space. In effect the one-dimensional process is represented by the simulated data $S(t)$ for the series t by: $S(t) = FFT^{-1}\left(\sqrt{\frac{P_{theor}(k_t)}{\Delta t}} \times FFT(\varepsilon_t)\right)$, where ε_t are the random numbers. In developing this report we performed the two-dimensional synthesis by Fourier transforming first in the vertical direction and then in the horizontal direction. $P_{theor}(k)$ for the vertical dimension is the "stretched" space PSD with unit variance, where we used $L_c = \overline{L_c} = 3.87$. For the horizontal dimension, we used the actual correlation length appropriate for that level. σ^2 was applied at the end of the process.

Using discrete Fourier transforms to estimate spectra, special care must be taken to avoid several well known pitfalls. The spacing between points must be kept small to avoid aliasing effects. Also the number of points simulated must be large so that numerical integration of the power spectrum in frequency space approximates the prescribed variance. On the other hand, since the method of AR synthesis provides the correct variance, a digital autoregressive (linear prediction) technique will avoid the FT variance problem. In addition, data generated by the AR process is "extendible" while that generated from Fourier synthesis is not. By "extendible" we mean that a given array can be made larger or "extended" beyond its present bounds by applying the AR process to data near the edges of the array. In applying the AR process, two new difficulties occur. First, one must decide how many AR coefficients to apply to achieve the desired statistics. Generally, the more coefficients that are used the better the desired PSD will be simulated. That is, an AR(p) process will exactly simulate the ACF to p lags. An increase in p translates into a better approximation of the remaining lags and therefore into less residual error of the PSD. In applying the AR process, the major difficulty is a need to overcome the filter relaxation time. If necessary this can be overcome by making several passes through the original white noise. Since it is advantageous to be able to continue a given structure array to a larger size, we set aside the Fourier technique and used instead a digital autoregressive (linear prediction) technique. The method has been described extensively in the literature[11] [12] and by us (see references 5-7). We simply state here that a sequence of values along each horizontal row and vertical column was generated from six autoregression coefficients, b_j, by the expression: $Y(J) = \varepsilon(J) - \sum_{j=1}^{6} b_j Y(J-j)$ where the b_j values were determined from a Levinson algorithm (see for example, reference 5).

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[11] Marple, S.L. (1987) *Digital Spectral Analysis with Applications*, Chapter 6, Prentice-Hall, Englewood Cliffs, New Jersey

[12] Kay, Steven M. (1988) *Modern Spectral Estimation, Theory & Application, Prentice-Hall, Englewood Cliffs, New Jersey*

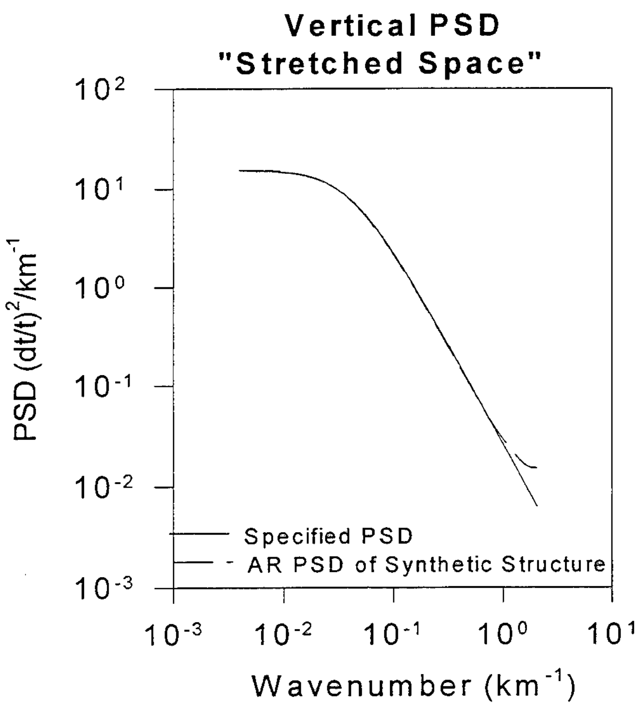

Figure 2. The solid curve depicts a prescribed "stretched" space vertical PSD having a Log-Log spectral slope of -2, $\overline{L_C} = 3.87$ km, and unit variance. The dashed curve depicts the autoregressively determined vertical PSD after generating vertical structure in the "stationary" vertical dimension.

A theoretical "stretched" space vertical PSD having a prescribed Log-Log spectral slope of -2, $\overline{L_C} = 3.87$, and unit variance is shown by the solid curve in Figure 2. This figure is typical of the graphs in this section that show Log-Log plots of PSD's measured in $\frac{(\delta\ temperature\ /\ temperature)^2}{wavenumber}$ and wavenumber measured in km^{-1}. The figures are obtained for vertical arrays having 8192 columns by 1024 rows with spacings of 0.1 km in the horizontal dimension and 0.244 km in the vertical dimension. These values translate to a horizontal range of 0-81.9 km and a vertical range of 50-300 km. A dashed curve in Figure 2 represents the autoregressively determined vertical PSD after generating vertical structure in the "stationary" vertical dimension. Here all vertical columns in the array were used to obtain the best estimate of the 6 autoregressive coefficients. Good agreement obtains between the prescribed PSD and the PSD of the synthetic vertical structure.

Remapping the stationary vertical structure in "stretched" space back to "normal" space, we obtain the desired non-stationary vertical structure. The solid curve in Figure 3 represents the theoretical vertical PSD at 51.2 km altitude where the prescribed vertical correlation length takes a value of L_C = 1.72 km and where the variance takes a value of 8.76×10^{-4}. Also plotted in Figure 3 is a dashed curve that represents the autoregressively determined vertical PSD after remapping back to the "normal" "non-

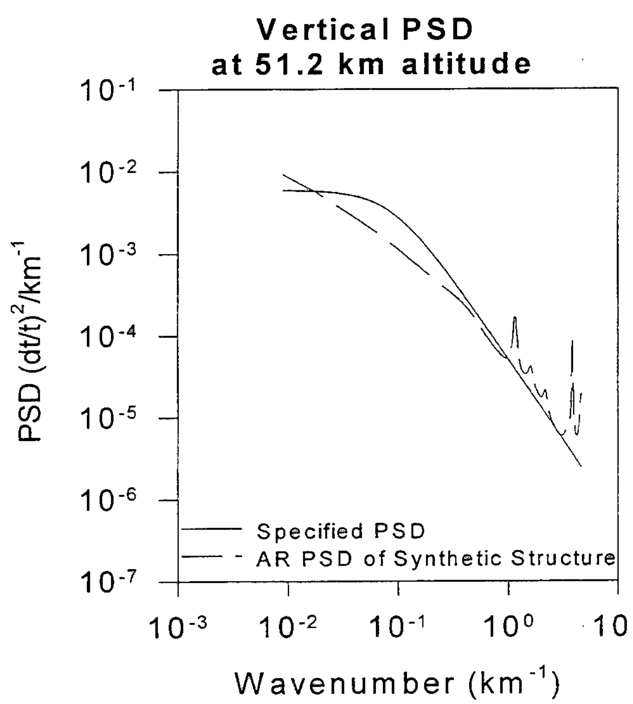

Figure 3. The solid curve depicts the theoretical vertical PSD at 51.2 km altitude where the prescribed vertical correlation length takes a value of L_C= 1.72 km and where the variance takes a value of 8.76×10^{-4}. The dashed curve plots the autoregressively determined vertical PSD after remapping back to the "normal" non stationary space.

stationary" space. In this space the structure varies with altitude, so only 11 horizontal rows of the array were used to obtain an estimate of the 6 autoregressive coefficients. Consequently the AR estimate of the vertical PSD is quite variable. In mapping back from the equally spaced stretched t points to the equally spaced vertical z points in "normal" space, linear interpolation was employed. The narrowest interpolations occurred at the lowest altitudes where $\Delta t = 0.244$ km corresponded to $\Delta z = 0.109$ km while the widest interpolations occurred at the highest altitudes where $\Delta t = 0.244$ km corresponded to $\Delta z = 0.632$ km. Thus the widest possible interpolations ranged from 0.122 km to 0.316 km at the lowest and highest altitudes respectively.

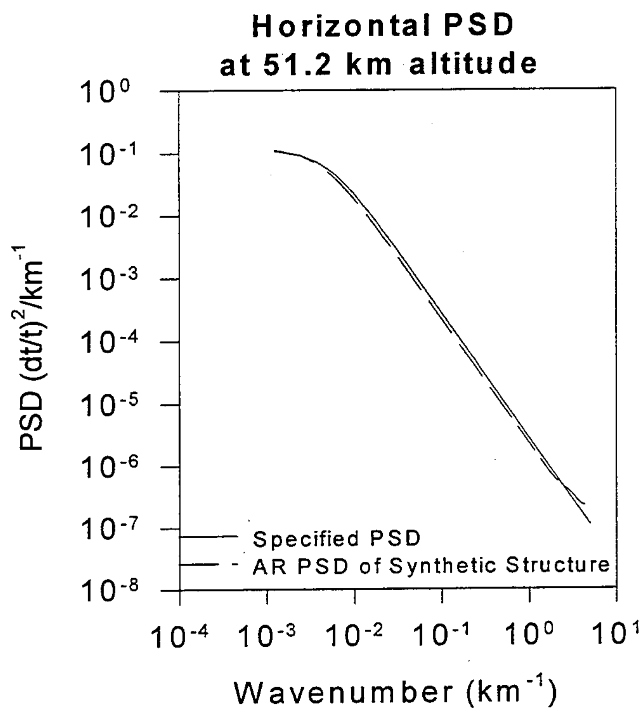

Figure 4. The solid curve shows a prescribed PSD for the structure in the horizontal dimension at 51.2 km altitude for a horizontal correlation length of $L_C = 32.9$ km. The dashed curve depicts the autoregressively determined horizontal PSD in normal space.

Figure 4 shows the prescribed and obtained PSD's for the structure in the horizontal dimension at 51.2 km altitude. The solid curve represents the theoretical PSD where the prescribed horizontal correlation length takes on a value of $L_C = 32.9$ km. The dashed curve represents the autoregressively determined horizontal PSD in normal space. Comparison of the two curves shows good agreement between the two.

Figure 5 depicts the prescribed (smooth curve) and obtained (variable curve) variances plotted against altitude. The spread in the variance is a function of the data spacing and number of points in a layer. It would decrease as the number of points increases or as the number of realizations increases. Said another way, the peaks, dips and apparent layering of the variance is artificial and due to the limited sample of transverse distance. The ragged curve would converge to the smooth curve as the transverse distance increases.

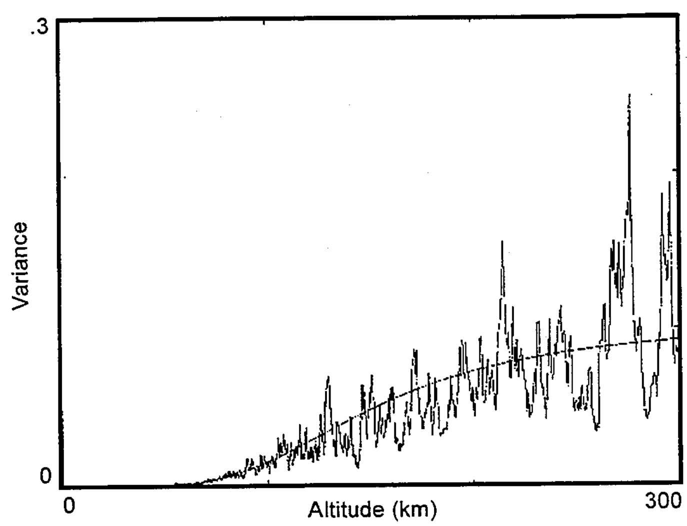

Figure 5. Prescribed (smooth curve) and obtained (variable curve) variance plotted as a function of altitude.

Figure 6 is a false color representation of the two-dimensional vertical sheet of non-stationary stochastic structure generated from the methods described above. The horizontal range is 0 - 819.2 km with a resolution of 1.6 km and the vertical range is 50 - 300 km with a resolution of 0. 714 km. Figure 6 reveals that the relative temperature fluctuation can exceed ±100% of the background temperature, which leads to some negative temperatures. This is due to the rather large values of prescribed temperature variance, assumed gaussian distribution, and realizations of large variances in the synthetic structure. For example Figure 5 shows a specified standard deviation, σ, of 0.3 at 260 km with a realized σ of about 0.4. Invoking the gaussian distribution of stochastic samples, -3σ values will produce negative temperatures. To avoid this effect, one could employ a different statistical distribution that is everywhere positive and provides additive properties. We note that the Chi Square distribution has these properties. Investigations are continuing to determine the appropriateness of using a different distribution.

5. CONCLUSION

Geophysical phenomena within a specified domain are often characterized by smooth continuous power spectral densities having a negative power law slope. A single pass stochastic one-dimensional autoregressive approach was employed to generate vertically correlated synthetic structure in "stretched space". The AR approach also was used to obtain synthetically correlated structure in the horizontal dimension. The joint goals of reducing the computational burden and of generating vertical sheets of non-stationary synthetic structure that is faithful to the prescribed descriptions were achieved. A complete two dimensional array consisting of 8192 horizontal by 1024 vertical points was generated on the Phillips Laboratory DEC Alpha architecture computer with an average execution time of 3 minutes. The process preserved the power spectral density law, correlation scale, variance, and probability density function.

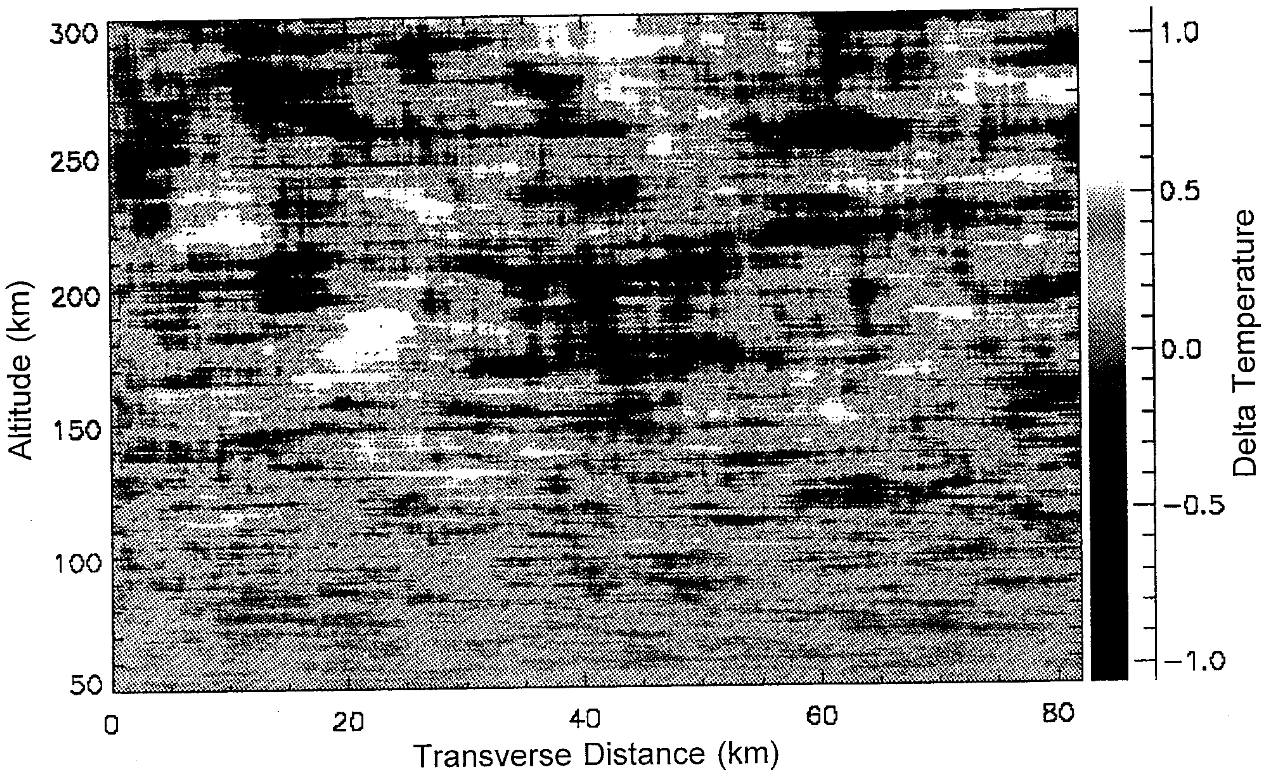

Figure 6. False color realization of two-dimensional vertical sheet of non-stationary stochastic structure.

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[1]Sharma, R.D., Gruninger, J.H., Sundberg, R.L., Duff, J.W., Bernstein, L.S., Robertson, D.C., Healey, R.J., (1991) Description of SHARC-2, The Strategic High Altitude Atmospheric Radiance Code, Phillips Laboratory technical report, PL-TR-91-2071, ADA 239 008

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